# Seminář 5

# The system $\lambda\mu$

- It is the system of *recursive types*.
- The recursive types come together with an equivalence relation $\approx$.
- The type assignment rules consist of the rules of $\lambda\rightarrow$ and the following rule

$$\frac{\Gamma \mid\!- M : \sigma \qquad \sigma \approx \sigma'}{\Gamma \mid\!- M : \sigma'}$$

## **The set T=**Type(**λμ**), **trees of types of λμ.**

(i) The set of types of **λμ, T** = Type(**λμ**), is defined by the following abstract grammar.

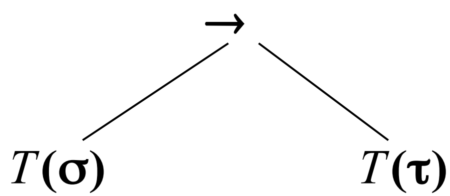
$$\mathbf{T} = \mathbf{V} \mid \mathbf{T} \rightarrow \mathbf{T} \mid \mu \mathbf{V}.\mathbf{T}$$

where **V** is the set of type variables.

(ii) Let $\sigma \in \mathbf{T}$ be a type. The *tree of* σ, $T(\sigma)$ is defined by induction on the structure of σ as follows:

$T(\alpha) = \alpha$ if $\alpha$ is a type variable

$T(\sigma \to \tau) =$



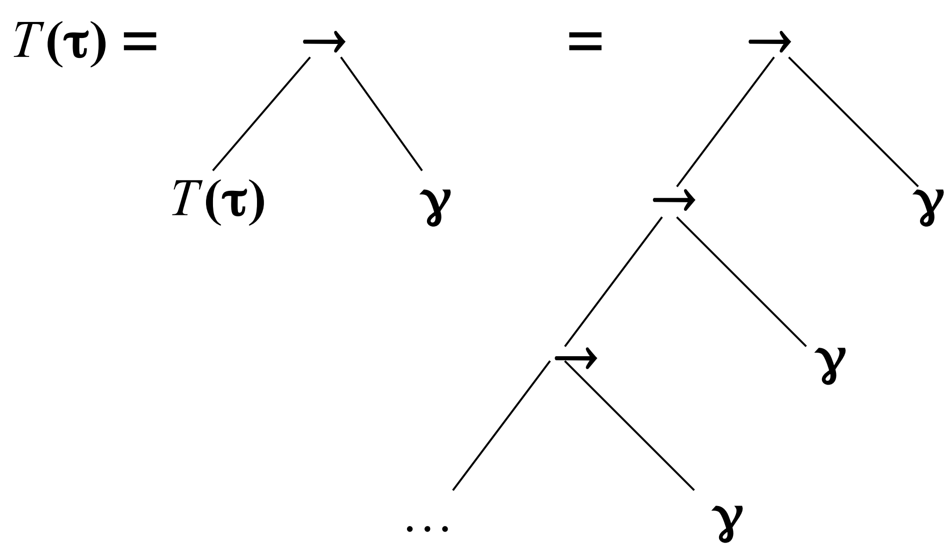

$$T(\mu\alpha.\sigma) = \begin{cases} \perp & \text{If } \sigma \equiv \mu\beta_1 \ldots \beta_n.\alpha \text{ for some } n \geq 0 \\ T(\sigma[\alpha := \mu\alpha.\sigma]) & \text{else} \end{cases}$$

(iii) The equivalence relation $\approx$ on trees is defined as follows:

$$\sigma \approx \tau \Leftrightarrow T(\sigma) = T(\tau)$$

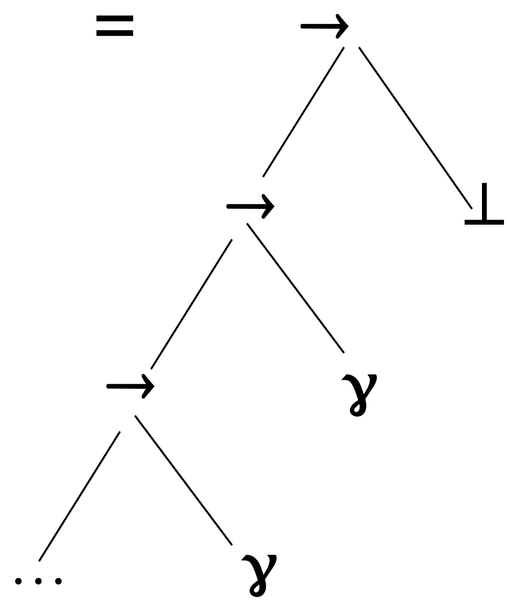
## Exercises.

(a) Assume $\tau \equiv \mu\alpha.\alpha \to \gamma$, then



(b) Assume $\boldsymbol{\tau \equiv (\mu\alpha.\alpha \to \gamma) \to \mu\delta\mu\beta.\beta}$, then

$T(\boldsymbol{\tau}) \quad = $



(c) $(\mu\alpha.\alpha \to \gamma) \approx (\mu\alpha(\alpha \to \gamma) \to \gamma)$.

(d) $\mu\alpha.\sigma \approx \sigma[\alpha := \mu\alpha.\sigma]$ for all $\sigma$, even if $\sigma \equiv \mu\beta.\alpha$.

# λμ

(start rule)

$$\frac{(x:\sigma)\in\Gamma}{\Gamma \mid\!- \ x:\sigma}$$

(→ - elimination)

$$\frac{\Gamma \mid\!- \ M:(\sigma\to\tau) \qquad \Gamma \mid\!- \ N:\sigma}{\Gamma \mid\!- \ (MN):\tau}$$

(→ - introduction)

$$\frac{\Gamma, x:\sigma \mid\!- \ M:\tau}{\Gamma \mid\!- \ (\lambda x.M):(\sigma\to\tau)}$$

(≈- rule)

$$\frac{\Gamma \mid\!- \ M:\sigma \qquad \sigma\approx\tau}{\Gamma \mid\!- \ M:\tau}$$

$\neg \mid\!- K : \alpha \to \alpha$

$\neg \mid\!- (\lambda x. (\lambda y.x)) : \alpha \to \alpha$